## **Empirical Study of Feature Selection Methods in Classification**

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#### Abstract

The use of feature selection can improve accuracy, efficiency, applicability and understandability of a learning process and the resulting learner. For this reason, many methods of automatic feature selection have been developed. By using the modularization of feature selection process, this paper evaluates a wide spectrum of these methods and some additional ones created by combination of different search and measure modules. The evaluation identifies the most interesting methods and shows some recommendations about which feature selection method should be used under different conditions.

#### 1 Introduction

The task of a classifier is to use feature vectors to assign the represented object to a category or class [10]. Feature selection help us to focus the attention of a classification algorithm in those features that are the most relevant to predict the class. Although theoretically, if the full statistical distribution were known, using more features could only improve results, in practical learning scenarios it may be better to use a reduced set of features [17]. Sometimes a large number of features in the input of induction algorithms may turn them very inefficient as memory and time consumers, even turning them inapplicable. Besides, irrelevant data may confuse algorithms making them reach false conclusions, and hence producing worse results.

Apart from increasing efficiency and applicability of classification algorithms, the costs of data acquisition may also be reduced when a smaller number of features is selected. In addition, the understandability of the results of a classification algorithm may be improved. Because of all those advantages feature selection has attracted much attention within the Machine Learning and Data Mining communities and many methods have been developed.

According to the different parts identified in feature selection methods [5, 16, 18], its process can be modular-

ized [2] as shown in figure 1. With this modularization almost every feature selection method can be characterized through the evaluation function and search strategy employed. By combining a set of evaluation functions and search strategies we can develop a map of feature selection. We did an extensive review of literature and gathered the most usual evaluation function as well as search strategies. This led to table 1. Our goal was to carry an extensive and rigorous empirical evaluation of the feature selection methods applied for classification.

Several reviews on feature selection can be found in the literature, for example, [5, 20, 21]. However, they are incomplete in the sense that the methods considered do not include every feasible cell on table 1. In addition, the set of classification problems considered is rather limited. Our goal was to include a wide spectrum of problems with varied number of features, classes, instances and nature of the features.



Figure 1. Feature selection process

#### Feature Selection methods for Classifica-2 tion

Our intention when addressing the endeavor stated in this paper was to develop a deep empirical study which could help us to gain some insights regarding the suitability of the different proposals in feature selection.

Having modularized the process of feature selection, we can take the two main factors (search and measure) and combine them creating a space of feature selection methods. The resulting combinations are represented in table 1.

All the feasible combinations are marked with a tick in the table and were selected for evaluation. Some of these combinations had already been proposed somewhere in the literature. To the best of our knowledge, those proposals have been recognized and a reference is included in the corresponding cell. Whenever no method for the combination has been found, we did created the corresponding method.

/	Search method		Com	olete		Seque	ential	Proba	bilistic	W	eta-heurisi	ic
Eval	uation function	Exhausti.	FOCUS 2	В&В	ABB	SFS	SBS	LVF	LVW	SA	GA	TS
	Basic consistency	Х		Х	Х	Х	Х	Х	Х	Х	Х	Х
<u> </u>	Inconsistent examples (Liu)	<b>&gt;</b> [9]	Х	>	<b>(</b> 9]	<b>&gt;</b> [9]	>	<b>&gt;</b> [9]	Х	>	~	>
sisno	Rough Set consistency	>	Х	>	>	>	>	>	Х	>	>	>
	Inconsistent example pairs (IEP)	>	Х	>	>	[5, 1]	>	>	Х	>	<	>
<i>\$.10</i>	Mutual information	>	Х	>	>	[26]	>	>	Х	>	~	>
әңі иори	Symmetrical uncertainty	>	Х	Х	Х	>	>	Х	>	>	>	>
nusofuI	Minimum description length criterion	[27]	Х	Х	Х	>	>	Х	>	>	<	>
92nateiU	Relief based feature set measure (RFSM)	>	Х	>	>	[3]	>	>	Х	>	>	>
әэиршлоfлә $_d$	Wrapper (using 10 fold cross validation)	>	X	Х	X	>	>	Х	>	>	>	>

# Table 1. Feature selection methods by search strategy and evaluation function

Unfeasible combinations are crossed in the table. They are not adequate because one of the following: i) basic consistency cannot guide any search (it is just a stop measure), ii) FOCUS2 can only use its own measure, iii) BB, ABB and LVF need monotonic measures, iv) LVW is designed for non-monotonic measures.

#### **3** Empirical methodology

With the goal stated in the section above in mind we designed and conducted an extensive and rigorous empirical study, out of which meaningful conclusions may be drawn. In this section we provided a detailed description of the experimental method followed, which certainly was hard work.

#### 3.1 Experimental design

Two are the main measures to be taken into account when evaluating a feature selection method: accuracy and feature reduction. Running time is also an important value to study, however, in this work we focus on accuracy and reduction of the number of features.

For every classification task, there are three main factors:

- 1. Feature selection method, with measure and search method as subfactors,
- 2. Learning algorithm that generates the classifier,
- 3. Classification problem represented in a data set.

The number of independent experiments is the number of the possible combinations of the three factors above, with some restrictions due to unacceptable high running time: i) feature selection methods that use complete search methods have been applied only to the data sets with less than 16 features and ii) methods with the wrapper measure has not been applied to the biggest dataset (adult).

Every experiment has been performed using 10 fold cross-validation. The result shown is always the average of the 10 folds.

To assess the significance of results, statistical tests have been used. Following the methodology recommended by Demsar [7] for this type of comparisons, we have used Friedman and Nemenyi statistical tests. A detailed description of these tests can be found in Zar's book [28].

#### 3.2 Data sets

We wanted to include in the study a wide range of classification problems. So we looked for problems in publicly available repositories seeking for representative problems in different properties. Finally, we have used 36 data sets. They are listed along with their main properties in table 2:

- *Data set* column show the name by which data sets are known.
- *Ex.* is the number of examples (tuples) in the data set.
- Feat. is the number of features.

- *Type* of features can be:
  - *Discr.*, all are discrete.
  - Cont., all are continuous.
  - Mixed, both types of features are present.
- *Cl.* is the number of classes.
- *Unk.* is the number of unknown values in the whole data set.

Dataset	Ex.	Feat.	Туре	Cl.	Unk.
adult	32561	14	Mixed	2	4262
anneal	898	38	Mixed	5	22175
audiology	226	69	Discr.	24	319
balance-scale	625	4	Discr.	3	0
breast-cancer	286	9	Mixed	2	9
bupa	345	6	Cont.	2	0
car	1728	6	Discr.	4	0
credit	690	15	Mixed	2	67
echocardiogram	131	10	Mixed	2	101
horse-colic	368	26	Mixed	2	1927
house-votes84	435	16	Discr.	2	392
ionosphere	351	32	Cont.	2	0
iris	150	4	Cont.	3	0
labor-neg	57	16	Mixed	2	326
led24	1200	24	Discr.	10	0
lenses	24	4	Discr.	3	0
lung-cancer	32	56	Discr.	3	5
lymphography	148	18	Discr.	4	0
mushrooms	8416	22	Discr.	2	2480
parity3+3	500	12	Discr.	2	0
pima	768	8	Cont.	2	0
post-operative	90	8	Mixed	3	3
primary-tumor	339	17	Discr.	21	225
promoters	106	57	Discr.	2	0
saheart	462	9	Mixed	2	0
shuttle-landing-	253	6	Discr.	2	0
control					
soybean	307	35	Discr.	19	712
splice	3190	60	Discr.	3	0
tic-tac-toe	958	9	Discr.	2	0
vehicle	846	18	Cont.	4	0
vowel	990	10	Cont.	11	0
wdbc	569	20	Cont.	2	0
wine	178	13	Cont.	3	0
yeast	1484	8	Cont.	10	0
yeast-class-RPR	186	79	Cont.	3	214
Z00	101	16	Discr.	7	0

#### Table 2. Data sets

All data sets have been taken from one of these sources:

• UCI: Classification Dataset repository at the University of California at Irvine, Irvine [14].

- ESL: Elements of Statistical Learning [12].
- Org: Dataset from the Orange web site [9].
- Sgi: Dataset repository from Silicon Graphics [11].
- Del: Delve (*Data for Evaluating Learning in Valid Experiments*) [23]

#### 3.3 Classifiers

In order to estimate the quality of feature selection performed by each method, the selected features are tested in a complete learning scenario of classification problems. The following well known learning methods [10] are considered. These methods have been chosen to cover the categories of methods most used.

To set up parameters of classifiers, preliminary experiments with different values were performed on the data sets. As we do not intend to compare learning methods, we just use a reasonable approach to get good results. The tests were done with all data sets using all features. The parameter values which performed better on average were chosen.

- Naive-Bayes (Nbayes). Despite its lower performance compared with other classifiers, Naive-Bayes is used in real problems with good results [25] and it can be successfully combined in bagging and boosting strategies. Nevertheless, the main reason why we have included this method is that, due to its simplicity, Naive-Bayes establishes a base on the minimal performance that other more elaborated methods should improve on.
- k Nearest Neighbors (kNN). This method has been considered as a representant of those methods that use distances in classification. After the preliminary experiments, the value k = 15 was chosen as a value large enough to get good results in all considered data sets.
- Classification trees [4] (C45). After testing different classification tree learners with the data sets, C4.5 [24] obtained better or similar results to the other tree learners. Besides, C4.5 is well known and commonly used to evaluate feature selectors. For these reasons, we intend this classifier to represent tree and rule based classifiers in our experiments.
- Artificial Neural Networks (ANN). There is a great diversity of ANN [13]. As a representation of ANN in classification we have chosen the well known multilayered perceptron. Preferring the simpler systems, just one hidden layer is used, as this is enough for universal approximation [15]. The number of nodes in the hidden layer is adjusted to each dataset by the simple criterion of taking the average between the number of

inputs and outputs. The network will have one output per class, and the class is decided by the output with the highest value. The training algorithm is standard back-propagation with learning rate of 0.1 and a maximum of 500 learning cycles.

#### **3.4** Data transformations

Some feature selection methods require certain conditions on data. Consequently, data are transformed just for these feature selection methods as described below. After feature selection, original data are passed to the learning methods.

For those methods that cannot cope with continuous features these are discretized using equal frequency intervals. More advanced discretization methods [19] were disregarded to avoid collateral effects in feature selection methods.

There are methods that only work on continuous features. For those, discrete features were translated to equidistant points in [0, 1].

For those methods that cannot cope with null or unknown values, these values were replaced by the average on continuous features or the most frequent value on discrete features.

#### 3.5 Development and running environment

The software used for learning methods has been Orange component-based data mining software [8], except for artificial neural networks where SNNS [22] was used integrated with OrangeSNNS package. All feature selection methods have been programmed using the Python programming language.

Experiments have run on a cluster of 8 nodes with one Xeon Dual 3.2Ghz processor and 1Gb of RAM.

#### 4 Experimental Results

The experiments described took long time to complete and generated a large amount of resulting data. An appropriate summarizing analysis is necessary to interpret them and achieve conclusions.

To help in this analysis, many tables and figures have been generated to compare feature selection methods. As an example, figure 2 shows the comparison of relevance measures using 'Sequential Backward Search'. The abscissa axis represents the ranking of each measure in relation with the others. The ranking is in the interval [1, n] when comparing n measures. The lower the ranking the better accuracy was achieved. The value shown for each measure is the average over the 36 data sets. Rows of figure 2 show the results for each of the four considered learning methods. In this way, we can compare the effect of feature selection on each learner and we assure independence for the application of statistical tests. The rectangle shows the Nemenyi critical distance at significance rate of p = 0.05 from the best method. Those methods outside the rectangle can be considered to obtain a worse accuracy.



#### Figure 2. Comparing accuracy with different relevance measures for SBS search method

Figure 3 shows the same comparison but on the number of features selected. The greater reduction the lower the ranking is. Only the reduction for one learning method is shown (C45). This is because the results are very similar as all measures are independent from the learner except Wrapper.

From these two figures we can see how Wrapper and MDL obtain significantly better results than most other measures. However, MDL fails in feature reduction, selecting more features in most cases, while Wrapper show a good compromise being the second in feature reduction. Symmetrical Uncertainty (SU) achieves the greatest reduction, but being the worse on accuracy. It can be concluded that Wrapper is the recommended measure when using Se-



Figure 3. Comparing number of features with different relevance measures for SBS search method

quential Backward Search (SBS).

Unfortunately, we can not simply deduce that the wrapper approach is the final solution as its main drawback is that its running time may be quite long. It is necessary to point out that precisely on SBS the first set of features to evaluate is the complete set —which includes all of them. And this may imply too long a time for a wrapper solution.

If results among learners are compared, we can see how in C45 results are more similar for all measures than within other learners. As this is a general observation on other figures, it may be concluded that feature selection influences C45 less than other learners. Intuitively, we can think that the reason is that C45 has its own feature selection embedded.



#### Figure 4. Comparing number of features with different search methods for mutual information relevance measure

Comparing measures with other search methods, the following facts can be observed. On complete searchbased methods, despite not finding significant differences —probably because only 21 data sets could be used— , Wrapper stands out with better accuracy and reduction when exhaustive search is used. On those complete searchbased methods that require monotonic measures, Liu and Information Gain leads to better results. Using Sequential Forward Search (SFS) we have not found significant differences on accuracy but IEP stands out on reduction. On randomized search methods, Wrapper leads on accuracy with no significance and SU leads on reduction followed by Wrapper, with significant differences with other methods in most cases.

Comparing search methods with each measure, we have not found statistically significant differences. However, SFS leads accuracy on all measures but Wrapper. This is quite interesting because this is a simple and efficient method. Using the Wrapper measure the best search methods are SBS and Exhaustive.

Mutual information (Inf) is a commonly used relevance measure in feature selection. As in the other measures, SFS leads accuracy but with no significative difference. Figure 4 shows the comparison according to feature reduction. GA can be rejected because of lower feature reduction while the other methods do not show significative difference for this measure.



Figure 5. Convex hull grouped by type of search method with results of C4.5

To provide a global view of results, as showing all methods in one figure will not be clear, their convex hulls are drawn instead. Figure 5 shows the convex hulls for the ranking results obtained with all methods for the C4.5 learner. Each convex hull is computed out of the ranking results for the feature selection algorithms grouped by its type of search method.

In this type of plots, the best methods would be near the origin of both axis since this means better accuracy and simultaneously higher feature reduction. In figure 5, we can see the contraposition of accuracy versus feature reduction as methods with better accuracy get worse reduction and vice versa

It is not a surprise that complete search group has the nearest frontier to origin. As these methods explore all feature sets, this is the expected result when evaluation functions provide appropriate information. Nevertheless, sequential search group has achieved pretty near results while having a better efficiency, that allows them being applied to much more data sets.

Because of the position of probabilistic and metaheuristic groups, we can see they have achieved worse results, specially on accuracy. Maybe the reason is that they need more iterations, more time than the other methods to get similar results. On the other hand, the advantage of these methods is that, in most cases, they can improve the results attained as more running time is allowed.

#### 5 Conclusions

An extensive and rigorous empirical study on feature selection methods for classification has been presented. To cover the spectrum of the great number of feature selection methods we have created and evaluated new methods by combination of evaluation functions with search strategies.

The experiments confirmed the usefulness of feature selection improving results in most of the problems while reducing the number of features used. The suspected contraposition of accuracy and feature reduction is also corroborated.

The wrapper approach is confirmed as the best option when it can be applied. In these cases, the recommended search strategy is Exhaustive or SBS. When wrapper is not applicable the results suggest using consistency measures or information gain. To achieve higher feature reduction the IEP measure can be used with SFS search.

These are only some recommendations that may be extracted from the empirical study. Applying some data mining or rule extraction techniques may lead us to an expert system that assist in finding the best method to use for each feature selection problem. As the results vary through learning methods, in future work it may be interesting to study the tolerance to irrelevant features of each learning method.

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